Derivation of equation (21)

$$P\{\hat{m}_p \geq m_p^* \big| i\} = \Phi_i^1 \left[\int_{-\infty}^{m_p^*} \left\{ \int_{m_p^*}^{\infty} g_M(\xi; m_p, \hat{\sigma}_{p,i}^R) d\xi \right\} g_P(m_p; m_p^*, \sigma_p) dm_p \right] \left[\begin{array}{l} \int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta; \bar{\delta}_i \rho_i) d\theta + \\ \int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta; \bar{\delta}_i \rho_i) d\theta \end{array} \right]$$
$$+ \Phi_i^1 \int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p} \left[\int_{-\infty}^{m_p^*} \left\{ \int_{m_p^*}^{\infty} g_M(\xi; m_p + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi \right\} g_P(m_p; m_p^*, \sigma_p) dm_p \right] h_i(\theta; \bar{\delta}_i \rho_i) d\theta$$

Because of the assumption that $\sigma_p / \hat{\sigma}_p \to 0$ we have $m_p^* = m_p$ and then:

$$\left[\int_{-\infty}^{m_p^*} \left\{ \int_{m_p^*}^{\infty} g_M(\xi; m_p, \hat{\sigma}_{p,i}^R) d\xi \right\} g_P(m_p; m_p^*, \sigma_p) dm_p \right] = \frac{1}{4}$$

Also,

$$\int_{-\infty}^{m_p^*} \left\{ \int_{m_p^*}^{\infty} g_M(\xi; m_p + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi \right\} g_P(m_p; m_p^*, \sigma_p) dm_p = \frac{1}{2} \int_{m_p^*}^{\infty} g_M(\xi; m_p^* + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi$$

Therefore

$$P\{\hat{m}_p \geq m_p^* \big| i\} = \frac{\Phi_i^1}{4} \left[\begin{array}{l} \int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta; \bar{\delta}_i \rho_i) d\theta + \\ \int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta; \bar{\delta}_i \rho_i) d\theta \end{array} \right] +$$
$$+ \frac{\Phi_i^1}{2} \int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p} \left\{ \int_{m_p^*}^{\infty} g_M(\xi; m_p^* + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta$$

$$\int_{m_p^*}^{\infty} g_M(\xi; m_p^* + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi = \frac{1}{\hat{\sigma}_p \sqrt{2\pi}} \int_{m_p^*}^{\infty} e^{-\frac{(m_p^* + \hat{\delta}_p^i(\theta) - \xi)^2}{2\hat{\sigma}_p^2}} d\xi$$

Let $t = \dfrac{(m_p^* + \hat{\delta}_p^i(\theta) - \xi)}{\sqrt{2}\hat{\sigma}_p}$ *Then* $dt = -\dfrac{d\xi}{\sqrt{2}\hat{\sigma}_p}$ *and* $\xi = (m_p^* + \hat{\delta}_p^i(\theta) - t\sqrt{2}\hat{\sigma}_p)$

Then

$$\int_{m_p^*}^{\infty} g_M(\xi; m_p^* + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi = -\frac{1}{\sqrt{\pi}} \int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{-\infty} e^{-t^2} dt =$$

$$= \begin{cases} \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty} e^{-t^2} dt = \dfrac{1}{2} erfc\left\{-\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \le 0 \\ \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty} e^{-t^2} dt = \dfrac{1}{\sqrt{\pi}} \left\{\int_{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{0} e^{-t^2} dt + \int_0^{\infty} e^{-t^2} dt\right\} = \dfrac{1}{\sqrt{\pi}} \left\{\int_0^{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}} e^{-t^2} dt + \int_0^{\infty} e^{-t^2} dt\right\} & \hat{\delta}_p^i(\theta) > 0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{2} erfc\left\{-\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \le 0 \\ \dfrac{1}{\sqrt{\pi}} \left\{\displaystyle\int_0^{\infty} e^{-t^2} dt - \int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty} e^{-t^2} dt + \int_0^{\infty} e^{-t^2} dt\right\} & \hat{\delta}_p^i(\theta) > 0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{2} erfc\left\{-\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \le 0 \\ \dfrac{1}{2} - \dfrac{1}{2} erfc\left\{\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} + \dfrac{1}{2} & \hat{\delta}_p^i(\theta) > 0 \end{cases}$$

$$\int_{m_p^*}^{\infty} g_M(\xi; m_p^* + \hat{\delta}_p^i(\theta), \hat{\sigma}_p) d\xi = \begin{cases} \dfrac{1}{2} erfc\left\{-\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \le 0 \\ 1 - \dfrac{1}{2} erfc\left\{\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) > 0 \end{cases}$$

$$P\{\hat{m}_p \ge m_p^* | i\} = \frac{\Phi_i^1}{4} \left[\begin{matrix} \displaystyle\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta; \bar{\delta}_i \rho_i) d\theta + \\ \displaystyle\int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta; \bar{\delta}_i \rho_i) d\theta \end{matrix}\right] + \frac{\Phi_i^1}{2} \int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p} \left\{ \begin{cases} \dfrac{1}{2} erfc\left\{-\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \le 0 \\ 1 - \dfrac{1}{2} erfc\left\{\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) > 0 \end{cases} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta \Bigg\}$$

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Derivation of the equations (22) and (23)

$$P\{\hat{m}_p \geq m_p^* | i\} = \frac{\Phi_i^1}{4}\left[\begin{array}{l}\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta + \\ \int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta\end{array}\right] + \frac{\Phi_i^1}{2}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left\{\left\{\begin{array}{ll}\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \leq 0 \\ 1-\frac{1}{2}erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) > 0\end{array}\right\} h_i(\theta;\bar{\delta}_i\rho_i)d\theta\right\}$$

$$\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\rho_i\sqrt{2\pi}}\int_{-\infty}^{-\tilde{\delta}_i^p} e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta$$

$$Let \quad t = \frac{(\theta-\bar{\delta}_i)}{\sqrt{2}\rho_i} \quad Then \quad dt = \frac{d\theta}{\sqrt{2}\rho_i} \quad and \quad \theta = (\bar{\delta}_i + t\sqrt{2}\rho_i)$$

$$\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\rho_i\sqrt{2\pi}}\int_{-\infty}^{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}} e^{-t^2}\sqrt{2}\rho_i dt$$

$$If \quad -\tilde{\delta}_i^p - \bar{\delta}_i \leq 0 \Rightarrow \bar{\delta}_i \geq -\tilde{\delta}_i^p$$

$$\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\sqrt{\pi}}\int_{\frac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}}^{\infty} e^{-t^2}dt = \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}$$

$$If \quad -\tilde{\delta}_i^p - \bar{\delta}_i \geq 0 \Rightarrow \tilde{\delta}_i^p \leq -\bar{\delta}_i$$

$$or \quad -\infty < \bar{\delta}_i < -\tilde{\delta}_i^p$$

$$\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}} e^{-t^2}dt = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{0} e^{-t^2}dt + \frac{1}{\sqrt{\pi}}\int_{0}^{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}} e^{-t^2}dt =$$

$$\frac{1}{2} + \left[\frac{1}{2} - \frac{1}{2}erfc\left\{-\frac{(\bar{\delta}_i+\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\}\right] = 1 - \frac{1}{2}erfc\left\{-\frac{(\bar{\delta}_i+\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\}$$

Thus

$$\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \left\{\begin{array}{ll}\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^p \leq \bar{\delta}_i < \infty \\ 1-\frac{1}{2}erfc\left\{-\frac{(\bar{\delta}_i+\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\} & -\infty < \bar{\delta}_i < -\tilde{\delta}_i^p\end{array}\right.$$

$$Similarly \int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\rho_i\sqrt{2\pi}}\int_{\tilde{\delta}_i^p}^{\infty} e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}} d\theta$$

$$\int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\rho_i\sqrt{2\pi}}\int_{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}}^{\infty} e^{-t^2}\sqrt{2}\rho_i dt$$

$$If \quad \tilde{\delta}_i^p - \bar{\delta}_i \le 0 \quad \Rightarrow \tilde{\delta}_i^p \le \bar{\delta}_i$$

$$\int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\sqrt{\pi}}\int_{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}}^{0} e^{-t^2}dt + \frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-t^2}dt$$

$$= \frac{1}{2} - \frac{1}{2}erfc\left\{-\frac{(\tilde{\delta}_i^p - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} + \frac{1}{2} = 1 - \frac{1}{2}erfc\left\{-\frac{(\tilde{\delta}_i^p - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}$$

$$If \quad \tilde{\delta}_i^p - \bar{\delta}_i \ge 0 \Rightarrow \tilde{\delta}_i^p \ge \bar{\delta}_i$$

$$\int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \frac{1}{\sqrt{\pi}}\int_{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}}^{\infty} e^{-t^2}dt = \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}$$

Thus,

$$\int_{-\infty}^{-\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \begin{cases} \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p + \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^p \le \bar{\delta}_i < \infty \\ 1 - \frac{1}{2}erfc\left\{-\frac{(\bar{\delta}_i + \tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\} & -\infty < \bar{\delta}_i < -\tilde{\delta}_i^p \end{cases}$$

$$\int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \begin{cases} 1 - \frac{1}{2}erfc\left\{-\frac{(\tilde{\delta}_i^p - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^p \le \bar{\delta}_i \\ \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^p \ge \bar{\delta}_i \end{cases}$$

Re*gions*

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Therefore

$$\left[\begin{array}{l}\int_{-\infty}^{\tilde{\delta}_i^p} h_i(\theta;\bar{\delta}_i\rho_i)d\theta \\ +\int_{\tilde{\delta}_i^p}^{\infty} h_i(\theta;\bar{\delta}_i\rho_i)d\theta\end{array}\right] = \begin{cases} \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^{\ p}+\bar{\delta}_i\ )}{\sqrt{2}\rho_i}\right\}+1-\frac{1}{2}erfc\left\{-\frac{(\tilde{\delta}_i^{\ p}-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^{\ p}\le\bar{\delta}_i\ <\infty \\ \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^{\ p}+\bar{\delta}_i\ )}{\sqrt{2}\rho_i}\right\}+\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^{\ p}-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^{\ p}\le\bar{\delta}_i\ <\tilde{\delta}_i^{p} \\ 1-\frac{1}{2}erfc\left\{-\frac{(\bar{\delta}_i+\tilde{\delta}_i^{\ p})}{\sqrt{2}\rho_i}\right\}+\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^{\ p}-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \bar{\delta}_i\ \le-\tilde{\delta}_i^{\ p}\end{cases}$$

Now:

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left\{\begin{array}{ll}\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)\le 0 \\ 1-\frac{1}{2}erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\end{array}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta = \int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left\{\begin{array}{ll}\frac{1}{2}erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta\le 0 \\ 1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta>0\end{array}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta =$$

$$=\int_{-\tilde{\delta}_i^p}^{0}\left\{\begin{array}{ll}\frac{1}{2}erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta\le 0 \\ 1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta>0\end{array}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta + \int_{0}^{\tilde{\delta}_i^p}\left\{\begin{array}{ll}\frac{1}{2}erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta\le 0 \\ 1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta>0\end{array}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta$$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left\{\begin{array}{ll}\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)\le 0 \\ 1-\frac{1}{2}erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\end{array}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta =$$

$\alpha_p^i > 0$

$$=\int_{-\tilde{\delta}_i^p}^{0}\left\{\frac{1}{2}erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta+\int_{0}^{\tilde{\delta}_i^p}\left\{1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta$$

$\alpha_p^i < 0$

$$=\int_{-\tilde{\delta}_i^p}^{0}\left\{1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta+\int_{0}^{\tilde{\delta}_i^p}\left\{\frac{1}{2}erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta$$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p} \left\{ \begin{matrix} \frac{1}{2} erfc\left\{ -\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p} \right\} & \hat{\delta}_p^i(\theta) \le 0 \\ 1 - \frac{1}{2} erfc\left\{ -\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p} \right\} & \hat{\delta}_p^i(\theta) > 0 \end{matrix} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta =$$

$$= \left\{ \begin{matrix} \int_{-\tilde{\delta}_i^p}^{0} \left\{ \frac{1}{2} erfc\left\{ -\frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta + \int_0^{\tilde{\delta}_i^p} \left\{ 1 - \frac{1}{2} erfc\left\{ \frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta & \alpha_p^i \ge 0 \\ \int_{-\tilde{\delta}_i^p}^{0} \left\{ 1 - \frac{1}{2} erfc\left\{ \frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta + \int_0^{\tilde{\delta}_i^p} \left\{ \frac{1}{2} erfc\left\{ -\frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta & \alpha_p^i < 0 \end{matrix} \right\}$$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p} \left\{ \begin{matrix} \frac{1}{2} erfc\left\{ -\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p} \right\} & \hat{\delta}_p^i(\theta) \le 0 \\ 1 - \frac{1}{2} erfc\left\{ -\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p} \right\} & \hat{\delta}_p^i(\theta) > 0 \end{matrix} \right\} h_i(\theta; \bar{\delta}_i \rho_i) d\theta =$$

$$= \left\{ \begin{matrix} \int_{-\tilde{\delta}_i^p}^{0} \left\{ \frac{1}{2} erfc\left\{ -\frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} \frac{e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}}{\rho_i\sqrt{2\pi}} d\theta + \int_0^{\tilde{\delta}_i^p} \left\{ 1 - \frac{1}{2} erfc\left\{ \frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} \frac{e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}}{\rho_i\sqrt{2\pi}} d\theta & \alpha_p^i \ge 0 \\ \int_{-\tilde{\delta}_i^p}^{0} \left\{ 1 - \frac{1}{2} erfc\left\{ \frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} \frac{e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}}{\rho_i\sqrt{2\pi}} d\theta + \int_0^{\tilde{\delta}_i^p} \left\{ \frac{1}{2} erfc\left\{ -\frac{\alpha_p^i \theta}{\sqrt{2}\hat{\sigma}_p} \right\} \right\} \frac{e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}}{\rho_i\sqrt{2\pi}} d\theta & \alpha_p^i < 0 \end{matrix} \right\}$$

$$Let \quad t=\frac{(\theta-\bar{\delta}_i\ )}{\sqrt{2}\rho_i} \quad Then \quad dt=\frac{d\theta}{\sqrt{2}\rho_i} \quad and \quad \theta=(\bar{\delta}_i\ +t\sqrt{2}\rho_i)$$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left.\begin{cases}\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)\le 0\\ 1-\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\end{cases}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta=$$

$$=\frac{1}{\sqrt{\pi}}\begin{cases}\int_{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}^{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}\left\{\frac{1}{2}erfc\left\{-\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt+\int_{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}^{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}\left\{1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt & \alpha_p^i\ge 0\\ \int_{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}^{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}\left\{1-\frac{1}{2}erfc\left\{\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt+\int_{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}^{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}\left\{\frac{1}{2}erfc\left\{-\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt & \alpha_p^i<0\end{cases}$$

Then

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left.\begin{cases}\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)\le 0\\ 1-\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\end{cases}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta=$$

$$=\frac{1}{2\sqrt{\pi}}\begin{cases}\int_{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}^{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}\left\{erfc\left\{-\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt+\int_{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}^{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}\left\{2-erfc\left\{\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt & \alpha_p^i\ge 0\\ \int_{\frac{(-\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}^{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}\left\{2-erfc\left\{\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt+\int_{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}^{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i\ )}{\sqrt{2}\rho_i}}\left\{erfc\left\{-\frac{\alpha_p^i(\bar{\delta}_i\ +t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\}e^{-t^2}dt & \alpha_p^i<0\end{cases}$$

Now $\int_a^\infty erfc\{a+bz\}\,e^{-z^2}dz$ cannot be integrated analytically.

Thus, recalling that

$$P\{\hat{m}_p\ge m_p^*|i\}=\frac{\Phi_i^1}{4}\left[\begin{matrix}\int_{-\infty}^{-\tilde{\delta}_i^p}h_i(\theta;\bar{\delta}_i\rho_i)d\theta+\\ \int_{\tilde{\delta}_i^p}^{\infty}h_i(\theta;\bar{\delta}_i\rho_i)d\theta\end{matrix}\right]+\frac{\Phi_i^1}{2}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left.\begin{cases}\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)\le 0\\ 1-\frac{1}{2}erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\end{cases}\right\}h_i(\theta;\bar{\delta}_i\rho_i)d\theta\Bigg\}$$

we have:

$$P\{\hat{m}_p \geq m_p^* | i\} = \frac{\Phi_i^1}{4} \left\{ \begin{array}{ll} \frac{1}{2} erfc\left\{\frac{(\tilde{\delta}_i^{\,p} + \bar{\delta}_i\,)}{\sqrt{2}\rho_i}\right\} + 1 - \frac{1}{2} erfc\left\{-\frac{(\tilde{\delta}_i^{\,p} - \bar{\delta}_i\,)}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^{\,p} \leq \bar{\delta}_i \; < \infty \\ \frac{1}{2} erfc\left\{\frac{(\tilde{\delta}_i^{\,p} + \bar{\delta}_i\,)}{\sqrt{2}\rho_i}\right\} + \frac{1}{2} erfc\left\{\frac{(\tilde{\delta}_i^{\,p} - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^{\,p} \leq \bar{\delta}_i \; < \tilde{\delta}_i^{\,p} \\ 1 - \frac{1}{2} erfc\left\{-\frac{(\bar{\delta}_i + \tilde{\delta}_i^{\,p})}{\sqrt{2}\rho_i}\right\} + \frac{1}{2} erfc\left\{\frac{(\tilde{\delta}_i^{\,p} - \bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \bar{\delta}_i \; \leq -\tilde{\delta}_i^{\,p} \end{array} \right\}$$

$$+ \frac{\Phi_i^1}{2} \frac{1}{2\sqrt{\pi}} \left\{ \begin{array}{ll} \int_{\frac{(-\tilde{\delta}_i^p - \bar{\delta}_i\,)}{\sqrt{2}\rho_i}}^{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}} \left\{ erfc\left\{-\frac{\alpha_p^i(\bar{\delta}_i \; + t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt + \int_{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}^{\frac{(\tilde{\delta}_i^p - \bar{\delta}_i\,)}{\sqrt{2}\rho_i}} \left\{ 2 - erfc\left\{\frac{\alpha_p^i(\bar{\delta}_i \; + t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt & \alpha_p^i \geq 0 \\ \int_{\frac{(-\tilde{\delta}_i^p - \bar{\delta}_i\,)}{\sqrt{2}\rho_i}}^{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}} \left\{ 2 - erfc\left\{\frac{\alpha_p^i(\bar{\delta}_i \; + t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt + \int_{\frac{-\bar{\delta}_i}{\sqrt{2}\rho_i}}^{\frac{(\tilde{\delta}_i^p - \bar{\delta}_i\,)}{\sqrt{2}\rho_i}} \left\{ erfc\left\{-\frac{\alpha_p^i(\bar{\delta}_i \; + t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt & \alpha_p^i < 0 \end{array} \right\}$$

which for the case of zero mean ($\bar{\delta}_i$ =0) reduces to:

$$P\{\hat{m}_p \geq m_p^* | i\} = \frac{\Phi_i^1}{4} \left[erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\}\right] \quad +$$

$$+ \frac{\Phi_i^1}{2} \frac{1}{2\sqrt{\pi}} \left\{ \begin{array}{ll} \int_{\frac{(-\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}}^{0} \left\{ erfc\left\{-\frac{\alpha_p^i(t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt + \int_0^{\frac{(\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}} \left\{ 2 - erfc\left\{\frac{\alpha_p^i(t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt & \alpha_p^i \geq 0 \\ \int_{\frac{(-\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}}^{0} \left\{ 2 - erfc\left\{\frac{\alpha_p^i(t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt + \int_0^{\frac{(\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}} \left\{ erfc\left\{-\frac{\alpha_p^i(t\sqrt{2}\rho_i)}{\sqrt{2}\hat{\sigma}_p}\right\}\right\} e^{-t^2} dt & \alpha_p^i < 0 \end{array} \right\}$$

$$P\{\hat{m}_p \geq m_p^* | i\} = \frac{\Phi_i^1}{4} \left[erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\}\right] \quad +$$

$$+ \frac{\Phi_i^1}{2} \frac{1}{2\sqrt{\pi}} \left\{ \begin{array}{ll} \int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} \left\{ erfc\left\{\frac{\alpha_p^i\sqrt{2}\rho_i}{\sqrt{2}\hat{\sigma}_p} t\right\}\right\} e^{-t^2} dt + \int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} \left\{ 2 - erfc\left\{\frac{\alpha_p^i\sqrt{2}\rho_i}{\sqrt{2}\hat{\sigma}_p} t\right\}\right\} e^{-t^2} dt & \alpha_p^i \geq 0 \\ \int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} \left\{ 2 - erfc\left\{-\frac{\alpha_p^i\sqrt{2}\rho_i}{\sqrt{2}\hat{\sigma}_p} t\right\}\right\} e^{-t^2} dt + \int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} \left\{ erfc\left\{-\frac{\alpha_p^i\sqrt{2}\rho_i}{\sqrt{2}\hat{\sigma}_p} t\right\}\right\} e^{-t^2} dt & \alpha_p^i < 0 \end{array} \right\}$$

Then

$$P\{\hat{m}_p \geq m_p^* \big| i\} = \frac{\Phi_i^1}{4} erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} \quad + \frac{\Phi_i^1}{2}\frac{1}{2\sqrt{\pi}}\left\{\begin{matrix} \int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} 2e^{-t^2} dt & \alpha_p^i \geq 0 \\ \int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} 2e^{-t^2} & \alpha_p^i < 0 \end{matrix}\right\} =$$

$$= \frac{\Phi_i^1}{4} erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} \quad + \frac{\Phi_i^1}{2}\frac{1}{\sqrt{\pi}}\int_0^{\frac{\tilde{\delta}_i^p}{\sqrt{2}\rho_i}} e^{-t^2} dt = \frac{\Phi_i^1}{4} erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{\Phi_i^1}{2}\left\{\frac{1}{2} - \frac{1}{2} erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\}\right\} =$$

$$= \frac{\Phi_i^1}{4} erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{\Phi_i^1}{4}\left\{1 - erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\}\right\} = \frac{\Phi_i^1}{4}$$

Derivation of equation (25)

$$P\{\hat{m}_p \geq m_p^* | \theta_1,\theta_2\} = \int_{-\infty}^{m_p^*} \left\{ \begin{array}{l} G(\theta_1,\theta_2|\theta_1,\theta_2)\int_{m_p^*}^{\infty} g_M(\xi;m_p,\hat{\sigma}_{p,i1,i2}^R)d\xi + \\ [1-G(\theta_1,\theta_2|\theta_1,\theta_2)] \\ \qquad \int_{m_p^*}^{\infty} g_M(\xi;m_p+\hat{\delta}_p^{i1,i2}(\theta_1,\theta_2),\hat{\sigma}_{p,m})d\xi \\ +G(\theta_1|\theta_1,\theta_2)\int_{m_p^*}^{\infty} g_M(\xi;m_p+\hat{\delta}_{p,m}^{i2}(\theta_2),\hat{\sigma}_{p,i1}^R)d\xi \\ +G(\theta_2|\theta_1,\theta_2)\int_{m_p^*}^{\infty} g_M(\xi;m_p+\hat{\delta}_p^{i1}(\theta_1),\hat{\sigma}_{p,i2}^R)d\xi \end{array} \right\} g_P(m_p;m_p^*,\sigma_p)dm_p$$

$$= \frac{1}{2}\left\{ \begin{array}{l} G(\theta_1,\theta_2|\theta_1,\theta_2)\int_{m_p^*}^{\infty} g_M(\xi;m_p^*,\hat{\sigma}_{p,i1,i2}^R)d\xi + \\ [1-G(\theta_1,\theta_2|\theta_1,\theta_2)] \\ \qquad \int_{m_p^*}^{\infty} g_M(\xi;m_p^*+\hat{\delta}_p^{i1,i2}(\theta_1,\theta_2),\hat{\sigma}_{p,m})d\xi \\ +G(\theta_1|\theta_1,\theta_2)\int_{m_p^*}^{\infty} g_M(\xi;m_p^*+\hat{\delta}_{p,m}^{i2}(\theta_2),\hat{\sigma}_{p,i1}^R)d\xi \\ +G(\theta_2|\theta_1,\theta_2)\int_{m_p^*}^{\infty} g_M(\xi;m_p^*+\hat{\delta}_p^{i1}(\theta_1),\hat{\sigma}_{p,i2}^R)d\xi \end{array} \right\}$$

But

$$\int_{m_p^*}^{\infty} g_M(\xi;m_p^*,\hat{\sigma}_{p,i1,i2}^R)d\xi = \frac{1}{2}$$

and

$$\int_{m_p^*}^{\infty} g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi = \begin{cases} \dfrac{1}{2}erfc\left\{-\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) \leq 0 \\ 1-\dfrac{1}{2}erfc\left\{\dfrac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta) > 0 \end{cases}$$

Then

$$P\{\hat{m}_p \geq m_p^* | \theta_1, \theta_2\} = \frac{1}{2}\left\{\begin{array}{l} G(\theta_1, \theta_2 | \theta_1, \theta_2)\frac{1}{2} + \\ [1 - G(\theta_1, \theta_2 | \theta_1, \theta_2)]\left\{\begin{array}{ll} \frac{1}{2} erfc\left\{-\frac{\hat{\delta}_p^{i1,i2}(\theta_1, \theta_2)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^{i1,i2}(\theta_1, \theta_2) \leq 0 \\ 1 - \frac{1}{2} erfc\left\{\frac{\hat{\delta}_p^{i1,i2}(\theta_1, \theta_2)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^{i1,i2}(\theta_1, \theta_2) > 0 \end{array}\right\} \\ + G(\theta_1 | \theta_1, \theta_2)\left\{\begin{array}{ll} \frac{1}{2} erfc\left\{-\frac{\hat{\delta}_{p,m}^{i2}(\theta_2)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_{p,m}^{i2}(\theta_2) \leq 0 \\ 1 - \frac{1}{2} erfc\left\{\frac{\hat{\delta}_{p,m}^{i2}(\theta_2)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_{p,m}^{i2}(\theta_2) > 0 \end{array}\right\} \\ + G(\theta_2 | \theta_1, \theta_2)\left\{\begin{array}{ll} \frac{1}{2} erfc\left\{-\frac{\hat{\delta}_{p,m}^{i1}(\theta_1)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_{p,m}^{i1}(\theta_1) \leq 0 \\ 1 - \frac{1}{2} erfc\left\{\frac{\hat{\delta}_{p,m}^{i1}(\theta_1)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_{p,m}^{i1}(\theta_1) > 0 \end{array}\right\} \end{array}\right\}$$

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Derivation of equations (35)-(39)

$$DEFL^1\big|i=\left.\begin{array}{l}\left[\int_{-\infty}^{m_p^*}K_S T\left\{\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p,\hat{\sigma}_{p,i}^R)d\xi\right\}g_P(m_p;m_p^*,\sigma_p)dm_p\right] \\ \hfill \left[\begin{array}{l}\int_{-\infty}^{-\tilde{\delta}_i^p}h_i(\theta;\bar{\delta}_i,\rho_i)d\theta+\\ \int_{\tilde{\delta}_i^p}^{\infty}h_i(\theta;\bar{\delta}_i,\rho_i)d\theta\end{array}\right] \\ +\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\int_{-\infty}^{m_p^*}K_S T\left\{\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right\}g_P(m_p;m_p^*,\sigma_p)dm_p\right] \\ \hfill h_i(\theta;\bar{\delta}_i,\rho_i)d\theta\end{array}\right\}$$

The first integral can be taken from the previous paper (Bagajewicz et al., 2004)

$$DEFL^1\big|i=\left.\begin{array}{l}DEFL^0\left[\dfrac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]\left[\int_{-\infty}^{-\tilde{\delta}_i^p}h_i(\theta;\bar{\delta}_i,\rho_i)d\theta+\int_{\tilde{\delta}_i^p}^{\infty}h_i(\theta;\bar{\delta}_i,\rho_i)d\theta\right] \\ +\dfrac{1}{2}K_S T\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left\{\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right\}h_i(\theta;\bar{\delta}_i,\rho_i)d\theta\end{array}\right\}$$

But since, (see above)

$$\left[\begin{array}{l}\int_{-\infty}^{\tilde{\delta}_i^p}h_i(\theta;\bar{\delta}_i\rho_i)d\theta\\ +\int_{\tilde{\delta}_i^p}^{\infty}h_i(\theta;\bar{\delta}_i\rho_i)d\theta\end{array}\right]=\begin{cases}\dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}+1-\dfrac{1}{2}erfc\left\{-\dfrac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^p\le\bar{\delta}_i<\infty\\ \dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}+\dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^p\le\bar{\delta}_i<\tilde{\delta}_i^p\\ 1-\dfrac{1}{2}erfc\left\{-\dfrac{(\bar{\delta}_i+\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\}+\dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \bar{\delta}_i\le-\tilde{\delta}_i^p\end{cases}$$

$$DEFL^1\big|i=\left.\begin{array}{l}DEFL^0\left[\dfrac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]\begin{cases}\dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}+1-\dfrac{1}{2}erfc\left\{-\dfrac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^p\le\bar{\delta}_i<\infty\\ \dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}+\dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^p\le\bar{\delta}_i<\tilde{\delta}_i^p\\ 1-\dfrac{1}{2}erfc\left\{-\dfrac{(\bar{\delta}_i+\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\}+\dfrac{1}{2}erfc\left\{\dfrac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \bar{\delta}_i\le-\tilde{\delta}_i^p\end{cases} \\ +\dfrac{1}{2}K_S T\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left\{\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right\}h_i(\theta;\bar{\delta}_i,\rho_i)d\theta\end{array}\right\}$$

$$\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi=\frac{1}{\hat{\sigma}_p\sqrt{2\pi}}\int_{-\infty}^{m_p^*}(m_p^*-\xi)e^{-\frac{(m_p^*+\hat{\delta}_p^i(\theta)-\xi)^2}{2\hat{\sigma}_p^2}}d\xi$$

$$Let\quad t=\frac{(-\xi+m_p^*+\hat{\delta}_p^i(\theta))}{\sqrt{2}\hat{\sigma}_p}\quad Then\quad dt=-\frac{d\xi}{\sqrt{2}\hat{\sigma}_p}\quad and\quad \xi=(m_p^*+\hat{\delta}_p^i(\theta)-t\sqrt{2}\hat{\sigma}_p)$$

Then

$$\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi=-\frac{1}{\sqrt{\pi}}\int_{\infty}^{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}(-\hat{\delta}_p^i(\theta)+t\sqrt{2}\hat{\sigma}_p)e^{-t^2}dt=$$

$$=\frac{1}{\sqrt{\pi}}\int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty}(-\hat{\delta}_p^i(\theta)+t\sqrt{2}\hat{\sigma}_p)e^{-t^2}dt=-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{\pi}}\int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty}e^{-t^2}dt+\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty}te^{-t^2}dt=$$

$$=-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{\pi}}\left\{\begin{array}{ll}\int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty}e^{-t^2}dt & \hat{\delta}_p^i(\theta)>0\\ \int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{0}e^{-t^2}dt+\int_0^{\infty}e^{-t^2}dt & \hat{\delta}_p^i(\theta)<0\end{array}\right\}+\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\left\{\begin{array}{ll}\int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{\infty}te^{-t^2}dt & \hat{\delta}_p^i(\theta)>0\\ \int_{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}^{0}te^{-t^2}dt+\int_0^{\infty}te^{-t^2}dt & \hat{\delta}_p^i(\theta)<0\end{array}\right\}$$

$$z=t^2\Rightarrow dz=2tdt$$

Then

$$\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi=-\left\{\begin{array}{ll}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}\hat{\delta}_p^i(\theta)\left[1-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}+1\right] & \hat{\delta}_p^i(\theta)<0\end{array}\right\}+$$

$$\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\left\{\begin{array}{ll}\frac{1}{2}\int_{\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}^{\infty}e^{-z}dz & \hat{\delta}_p^i(\theta)>0\\ -\int_0^{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}}te^{-t^2}dt+\frac{1}{2}\int_0^{\infty}e^{-z}dz & \hat{\delta}_p^i(\theta)<0\end{array}\right\}$$

$$\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi=-\begin{cases}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}\hat{\delta}_p^i(\theta)\left[1-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}+1\right] & \hat{\delta}_p^i(\theta)<0\end{cases}+$$

$$+\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\begin{cases}\frac{1}{2}\left[-e^{-z}\right]_{\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}^{\infty} & \hat{\delta}_p^i(\theta)>0\\ -\frac{1}{2}\int_0^{\left[-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}e^{-z}dz+\frac{1}{2}\left[-e^{-z}\right]_0^{\infty} & \hat{\delta}_p^i(\theta)<0\end{cases}$$

$$\left[-e^{-z}\right]_{\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}^{\infty}=e^{-\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}$$

$$\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi=-\begin{cases}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}\hat{\delta}_p^i(\theta)\left[1-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}+1\right] & \hat{\delta}_p^i(\theta)<0\end{cases}$$

$$+\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\begin{cases}\frac{1}{2}e^{-\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2} & \hat{\delta}_p^i(\theta)>0\\ -\frac{1}{2}\left[-e^{-z}\right]_0^{\left[-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}+\frac{1}{2} & \hat{\delta}_p^i(\theta)<0\end{cases}=-\begin{cases}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}\hat{\delta}_p^i(\theta)\left[2-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}\right] & \hat{\delta}_p^i(\theta)<0\end{cases}$$

$$+\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\begin{cases}\frac{1}{2}e^{-\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2} & \hat{\delta}_p^i(\theta)>0\\ -\frac{1}{2}\left[-e^{-\left[-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}+1\right]+\frac{1}{2} & \hat{\delta}_p^i(\theta)<0\end{cases}=-\begin{cases}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}\hat{\delta}_p^i(\theta)\left[2-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}\right] & \hat{\delta}_p^i(\theta)<0\end{cases}$$

$$+\frac{\sqrt{2}\hat{\sigma}_p}{\sqrt{\pi}}\begin{cases}\frac{1}{2}e^{-\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}e^{-\left[-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2} & \hat{\delta}_p^i(\theta)<0\end{cases}=-\begin{cases}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{1}{2}\hat{\delta}_p^i(\theta)\left[2-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}\right] & \hat{\delta}_p^i(\theta)<0\end{cases}+\frac{\sqrt{2}\hat{\sigma}_p}{2\sqrt{\pi}}e^{-\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}$$

Then:

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right]h_i(\theta,\rho_i)d\theta =$$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\frac{\hat{\sigma}_p}{\sqrt{2\pi}}e^{-\left[\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right]^2}-\begin{cases}\frac{1}{2}\hat{\delta}_p^i(\theta)erfc\left\{\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\} & \hat{\delta}_p^i(\theta)>0\\ \frac{\hat{\delta}_p^i(\theta)}{2}\left[2-erfc\left\{-\frac{\hat{\delta}_p^i(\theta)}{\sqrt{2}\hat{\sigma}_p}\right\}\right] & \hat{\delta}_p^i(\theta)<0\end{cases}\right]h_i(\theta;\bar{\delta}_i,\rho_i)d\theta$$

Thus

Substituting $\hat{\delta}_p^i(\theta)=\alpha_p^i\theta$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right]h_i(\theta;\bar{\delta}_i,\rho_i)d\theta =$$

$$=\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\frac{\hat{\sigma}_p}{\sqrt{2\pi}}e^{-\left[\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right]^2}-\begin{cases}\frac{\alpha_p^i\theta}{2}erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\} & \alpha_p^i\theta>0\\ \frac{\alpha_p^i\theta}{2}\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right] & \alpha_p^i\theta<0\end{cases}\right]e^{-(\theta-\bar{\delta}_i)^2/(2\rho_i^2)}d\theta$$

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right]h_i(\theta;\bar{\delta}_i,\rho_i)d\theta=\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\frac{\hat{\sigma}_p}{\sqrt{2\pi}}e^{-\left[\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right]^2}e^{-(\theta-\bar{\delta}_i)^2/(2\rho_i^2)}d\theta-$$

$$-\begin{cases}\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{0}\theta\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta+\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta\, erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta & \alpha_p^i>0\\ \frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{0}\theta erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta+\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta & \alpha_p^i<0\end{cases}$$

But

$$\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\frac{\hat{\sigma}_p}{\sqrt{2\pi}}e^{-\left[\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right]^2}e^{-(\theta-\bar{\delta}_i)^2/(2\rho_i^2)}d\theta=\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}e^{-\frac{\left[\alpha_p^i\theta\right]^2}{2\hat{\sigma}_p^2}}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta$$

$$a\theta^2+b(\theta-\bar{\delta}_i)^2=r(\theta-e)^2+f\Rightarrow(a+b)\theta^2-2b\theta\bar{\delta}_i+b\bar{\delta}_i^2=r\theta^2-2re\theta+re^2+f$$

$$(a+b)=r\qquad 2b\bar{\delta}_i=2re\qquad b\bar{\delta}_i^2=re^2+f$$

$$\Rightarrow b\bar{\delta}_i=(a+b)e\Rightarrow e=b\bar{\delta}_i/(a+b)\Rightarrow f=b\bar{\delta}_i^2-re^2=b\bar{\delta}_i^2-(a+b)b^2\bar{\delta}^2{}_i/(a+b)^2$$

$$r=\frac{\left[\alpha_p^i\right]^2}{2\hat{\sigma}_p^2}+\frac{1}{2\rho_i^2}\qquad e=\frac{\bar{\delta}_i}{2\rho_i^2}\frac{1}{\left[\frac{\left[\alpha_p^i\right]^2}{2\hat{\sigma}_p^2}+\frac{1}{2\rho_i^2}\right]}\quad f=\frac{1}{2\rho_i^2}\bar{\delta}_i^2-\frac{\bar{\delta}_i^2}{4\rho_i^4}\frac{1}{\left[\frac{\left[\alpha_p^i\right]^2}{2\hat{\sigma}_p^2}+\frac{1}{2\rho_i^2}\right]}$$

$$\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}e^{-\frac{\left[\alpha_p^i\theta\right]^2}{2\hat{\sigma}_p}}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta=\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-f}}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}e^{-r(\theta-e)^2}d\theta$$

$$z=\sqrt{r}(\theta-e);dz=\sqrt{r}\;d\theta$$

$$\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}e^{-\frac{\left[\alpha_p^i\theta\right]^2}{2\hat{\sigma}_p}}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta=\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-f}}{\sqrt{2\pi}\rho_i\sqrt{r}}\int_{\sqrt{r}(-\tilde{\delta}_i^p-e)}^{\sqrt{r}(\tilde{\delta}_i^p-e)}e^{-z^2}dz$$

$$=\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-f}}{\sqrt{2\pi}\rho_i\sqrt{r}}\frac{\sqrt{\pi}}{2}\left[\frac{2}{\sqrt{\pi}}\int_{\sqrt{r}(-\tilde{\delta}_i^p-e)}^{\sqrt{r}(\tilde{\delta}_i^p-e)}e^{-z^2}dz\right]=$$

$$\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p} e^{-\frac{\left[\alpha_p^i\theta\right]^2}{2\hat{\sigma}_p}} e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}\,d\theta = \frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-f}}{\sqrt{2\pi}\rho_i\sqrt{r}}\int_{\sqrt{r}(-\tilde{\delta}_i^p-e)}^{\sqrt{r}(\tilde{\delta}_i^p-e)} e^{-z^2}\,dz =$$

$$\frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-f}}{\sqrt{2\pi}\rho_i\sqrt{r}}\frac{\sqrt{\pi}}{2}\left[\frac{2}{\sqrt{\pi}}\int_{\sqrt{r}(-\tilde{\delta}_i^p-e)}^{\sqrt{r}(\tilde{\delta}_i^p-e)} e^{-z^2}\,dz\right] =$$

$$= \frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-f}}{\sqrt{2\pi}\rho_i\sqrt{r}}\frac{\sqrt{\pi}}{2}\begin{cases}\frac{2}{\sqrt{\pi}}\int_{\sqrt{r}(-\tilde{\delta}_i^p-e)}^{0} e^{-z^2}\,dz + \frac{2}{\sqrt{\pi}}\int_{0}^{\sqrt{r}(\tilde{\delta}_i^p-e)} e^{-z^2}\,dz & -\tilde{\delta}_i^p - e \le 0; \tilde{\delta}_i^p - e \ge 0 \\ & \Rightarrow -\tilde{\delta}_i^p \le e \le \tilde{\delta}_i^p \\ \frac{2}{\sqrt{\pi}}\int_{\sqrt{r}(-\tilde{\delta}_i^p-e)}^{\sqrt{r}(\tilde{\delta}_i^p-e)} e^{-z^2}\,dz & -\tilde{\delta}_i^p - e \ge 0; \tilde{\delta}_i^p - e \ge 0 \\ & \Rightarrow -\tilde{\delta}_i^p \ge e \\ \frac{2}{\sqrt{\pi}}\int_{\sqrt{r}(-e-\tilde{\delta}_i^p)}^{\sqrt{r}(-e+\tilde{\delta}_i^p)} e^{-z^2}\,dz = \frac{2}{\sqrt{\pi}}\int_{\sqrt{r}(e+\tilde{\delta}_i^p)}^{\sqrt{r}(e-\tilde{\delta}_i^p)} e^{-z^2}\,dz & -\tilde{\delta}_i^p - e \le 0; \tilde{\delta}_i^p - e \le 0 \\ & \Rightarrow e \ge \tilde{\delta}_i^p\end{cases}$$

$$r = \frac{\left[\alpha_p^i\right]^2}{2\hat{\sigma}_p^2} + \frac{1}{2\rho_i^2} = \frac{1}{2\hat{\sigma}_p^2\rho_i^2}\left[\left[\alpha_p^i\right]^2\rho_i^2 + \hat{\sigma}_p^2\right] \qquad e = \frac{\bar{\delta}_i}{2\rho_i^2}\frac{1}{\left[\frac{\left[\alpha_p^i\right]^2}{2\hat{\sigma}_p^2} + \frac{1}{2\rho_i^2}\right]} = \frac{\bar{\delta}_i}{\rho_i^2}\frac{\hat{\sigma}_p^2\rho_i^2}{\left[\left[\alpha_p^i\right]^2\rho_i^2 + \hat{\sigma}_p^2\right]} = \frac{\bar{\delta}_i}{\rho_i^2}\frac{\hat{\sigma}_p^2\rho_i^2}{2\hat{\sigma}_p^2\rho_i^2 r};$$

$$e = \frac{\bar{\delta}_i}{2\rho_i^2 r} \qquad f = \frac{1}{2\rho_i^2}\bar{\delta}_i^2 - \frac{\bar{\delta}_i^2}{4\rho_i^4}\frac{1}{\left[\frac{\left[\alpha_p^i\right]^2}{2\hat{\sigma}_p^2} + \frac{1}{2\rho_i^2}\right]} = \frac{\bar{\delta}_i^2}{2\rho_i^2}\left(1 - \frac{1}{2\rho_i^2 r}\right)$$

$$= \frac{\hat{\sigma}_p}{\sqrt{2\pi}}\frac{e^{-\frac{\bar{\delta}_i^2}{2\rho_i^2}\left(1-\frac{1}{2\rho_i^2 r}\right)}}{\sqrt{2\pi}\rho_i\sqrt{r}}\frac{\sqrt{\pi}}{2}\begin{cases}1 - erfc\left\{\sqrt{r}(e+\tilde{\delta}_i^p)\right\} + 1 - erfc\left\{\sqrt{r}(\tilde{\delta}_i^p - e)\right\} & -\tilde{\delta}_i^p \le e \le \tilde{\delta}_i^p \\ erfc\left\{\sqrt{r}(-\tilde{\delta}_i^p - e)\right\} - erfc\left\{\sqrt{r}(\tilde{\delta}_i^p - e)\right\} & -\tilde{\delta}_i^p \ge e \\ erfc\left\{\sqrt{r}(e-\tilde{\delta}_i^p)\right\} - erfc\left\{\sqrt{r}(e+\tilde{\delta}_i^p)\right\} & e \ge \tilde{\delta}_i^p\end{cases}$$

Then

$$\int_{-\tilde{\delta}_i^p}^{\tilde{\delta}_i^p}\left[\int_{-\infty}^{m_p^*}(m_p^*-\xi)g_M(\xi;m_p^*+\hat{\delta}_p^i(\theta),\hat{\sigma}_p)d\xi\right]h_i(\theta;\bar{\delta}_i,\rho_i)d\theta =$$

$$=\frac{\hat{\sigma}_p e^{-\frac{\bar{\delta}_i^2}{2\rho_i^2}(1-\frac{1}{2\rho_i^2 r})}}{4\rho_i\sqrt{\pi r}}\begin{cases}1-erfc\left\{\sqrt{r}(e+\tilde{\delta}_i^p)\right\}+1-erfc\left\{\sqrt{r}(\tilde{\delta}_i^p-e)\right\} & -\tilde{\delta}_i^p\le e\le\tilde{\delta}_i^p\\ erfc\left\{\sqrt{r}(-\tilde{\delta}_i^p-e)\right\}-erfc\left\{\sqrt{r}(\tilde{\delta}_i^p-e)\right\} & -\tilde{\delta}_i^p\ge e\\ erfc\left\{\sqrt{r}(e-\tilde{\delta}_i^p)\right\}-erfc\left\{\sqrt{r}(e+\tilde{\delta}_i^p)\right\} & e\ge\tilde{\delta}_i^p\end{cases}$$

$$-\begin{cases}\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{0}\theta\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta+\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta\, erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta & \alpha_p^i>0\\ \frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{0}\theta erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta+\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta & \alpha_p^i<0\end{cases}$$

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]\left\{\begin{array}{l}\begin{cases}\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}+1-\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \tilde{\delta}_i^p\le\bar{\delta}_i<\infty\\ \frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p+\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\}+\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & -\tilde{\delta}_i^p\le\bar{\delta}_i<\tilde{\delta}_i^p\\ 1-\frac{1}{2}erfc\left\{-\frac{(\bar{\delta}_i+\tilde{\delta}_i^p)}{\sqrt{2}\rho_i}\right\}+\frac{1}{2}erfc\left\{\frac{(\tilde{\delta}_i^p-\bar{\delta}_i)}{\sqrt{2}\rho_i}\right\} & \bar{\delta}_i\le-\tilde{\delta}_i^p\end{cases}\\ +\frac{1}{2}K_S T\frac{\hat{\sigma}_p e^{-\frac{\bar{\delta}_i^2}{2\rho_i^2}(1-\frac{1}{2\rho_i^2 r})}}{4\rho_i\sqrt{\pi r}}\begin{cases}1-erfc\left\{\sqrt{r}(e+\tilde{\delta}_i^p)\right\}+1-erfc\left\{\sqrt{r}(\tilde{\delta}_i^p-e)\right\} & -\tilde{\delta}_i^p\le e\le\tilde{\delta}_i^p\\ erfc\left\{\sqrt{r}(-\tilde{\delta}_i^p-e)\right\}-erfc\left\{\sqrt{r}(\tilde{\delta}_i^p-e)\right\} & -\tilde{\delta}_i^p\ge e\\ erfc\left\{\sqrt{r}(e-\tilde{\delta}_i^p)\right\}-erfc\left\{\sqrt{r}(e+\tilde{\delta}_i^p)\right\} & e\ge\tilde{\delta}_i^p\end{cases}\\ -\frac{1}{2}K_S T\begin{cases}\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{0}\theta\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta+\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta\, erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta & \alpha_p^i>0\\ \frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_{-\tilde{\delta}_i^p}^{0}\theta erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta+\frac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta\left[2-erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{(\theta-\bar{\delta}_i)^2}{2\rho_i^2}}d\theta & \alpha_p^i<0\end{cases}\end{array}\right\}$$

For the mean=0, we have, *e*=*f*=0. Then

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right] erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{1}{2}K_S T\,\frac{\hat{\sigma}_p}{4\rho_i\sqrt{\pi r}}\left[2 - 2erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right]$$
$$\left. - \frac{1}{2}K_S T\begin{cases}\dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{-\tilde{\delta}_i^p}^{0}\theta\left[2 - erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{\theta^2}{2\rho_i^2}}d\theta + \dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_0^{\tilde{\delta}_i^p}\theta\, erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i > 0\\ \dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{-\tilde{\delta}_i^p}^{0}\theta erfc\left\{\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}e^{-\frac{\theta^2}{2\rho_i^2}}d\theta + \dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_0^{\tilde{\delta}_i^p}\theta\left[2 - erfc\left\{-\frac{\alpha_p^i\theta}{\sqrt{2}\hat{\sigma}_p}\right\}\right]e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i < 0\end{cases}\right\}$$

Then

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right] erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{1}{2}K_S T\,\frac{\hat{\sigma}_p}{2\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right]$$
$$\left. - \frac{1}{2}K_S T\begin{cases}\dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{-\tilde{\delta}_i^p}^{0}2\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i > 0\\ \dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{-\tilde{\delta}_i^p}^{0}2\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i < 0\end{cases}\right\} =$$

$$DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right] erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{K_S T}{2}\,\frac{\hat{\sigma}_p}{2\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] - \frac{K_S T}{2}\begin{cases}\dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{-\tilde{\delta}_i^p}^{0}2\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i > 0\\ \dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{0}^{\tilde{\delta}}2\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i < 0\end{cases}$$

Then

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right] erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{K_S T}{2}\,\frac{\hat{\sigma}_p}{2\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] + \frac{K_S T}{2}\begin{cases}\dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{0}^{\tilde{\delta}_i^p}2\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i > 0\\ \dfrac{\alpha_p^i}{2\sqrt{2\pi}\rho_i}\displaystyle\int_{0}^{\tilde{\delta}_i^p}2\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta & \alpha_p^i < 0\end{cases}$$

or

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right] erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{K_S T}{2}\,\frac{\hat{\sigma}_p}{2\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] + \frac{K_S T}{2}\,\frac{\left|\alpha_p^i\right|}{\sqrt{2\pi}\rho_i}\int_0^{\tilde{\delta}_i^p}\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta$$

$$z = \theta^2;\, dz = 2\theta d\theta$$

$$\int_0^{\tilde{\delta}_i^p}\theta e^{-\frac{\theta^2}{2\rho_i^2}}d\theta = \frac{1}{2}\int_0^{\left(\tilde{\delta}_i^p\right)^2}e^{-\frac{z}{2\rho_i^2}}dz = \frac{1}{2}\left[-2\rho_i^2 e^{-\frac{z}{2\rho_i^2}}\right]_0^{\left(\tilde{\delta}_i^p\right)^2} = -\rho_i^2 e^{-\frac{\left(\tilde{\delta}_i^p\right)^2}{2\rho_i^2}} + \rho_i^2 = \rho_i^2(1 - e^{-\frac{\left(\tilde{\delta}_i^p\right)^2}{2\rho_i^2}})$$

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{K_S T}{2}\frac{\hat{\sigma}_p}{2\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] + \frac{K_S T}{2}\frac{\left|\alpha_p^i\right|}{\sqrt{2\pi}\rho_i}\rho_i^2\left[1 - e^{-\frac{\left(\tilde{\delta}_i^{\,p}\right)^2}{2\rho_i^2}}\right]$$

We use $DEFL^0 = \gamma K_S T\hat{\sigma}_p$

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{DEFL^0}{2\gamma}\frac{1}{2\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] + \frac{DEFL^0}{2\gamma\hat{\sigma}_p}\frac{\left|\alpha_p^i\right|}{\sqrt{2\pi}\rho_i}\rho_i^2\left[1 - e^{-\frac{\left(\tilde{\delta}_i^{\,p}\right)^2}{2\rho_i^2}}\right]\Bigg\} =$$

$$DEFL^1\Big|i = DEFL^0\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{DEFL^0}{4\gamma\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] + \frac{DEFL^0\left|\alpha_p^i\right|\rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}}\left[1 - e^{-\frac{\left(\tilde{\delta}_i^{\,p}\right)^2}{2\rho_i^2}}\right]\Bigg\} =$$

$$DEFL^1\Big|i = DEFL^0\left\{\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\} + \frac{1}{4\gamma\rho_i\sqrt{\pi r}}\left[1 - erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right] + \frac{\left|\alpha_p^i\right|\rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}}\left[1 - e^{-\frac{\left(\tilde{\delta}_i^{\,p}\right)^2}{2\rho_i^2}}\right]\right\}\Bigg\}$$

Derivation of equations (42)-(43)

$$DEFL^1\Big|i = DEFL^0\left\{\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\}+\frac{1}{4\gamma\rho_i\sqrt{\pi r}}\left[1-erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right]-\frac{\alpha_p^i\rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}}\left[1-e^{-\frac{\left(\tilde{\delta}_i^{\,p}\right)^2}{2\rho_i^2}}\right]\right\}$$

$$\left.\begin{array}{l}
\rho_i << \tilde{\delta}_i^{\,p} \Rightarrow x=\dfrac{\tilde{\delta}_i^{\,p}}{\rho_i} >> 1 \\
DEFL^1\Big|i = DEFL^0\left\{\left[\dfrac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\dfrac{x}{\sqrt{2}}\right\}+\dfrac{1}{4\gamma\rho_i\sqrt{\pi r}}\left[1-erfc\left\{\rho_i\sqrt{r}x\right\}\right]-\dfrac{\alpha_p^i\rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}}\left[1-e^{-\frac{x^2}{2}}\right]\right\}
\end{array}\right\}$$

$$erfc(x)=1-\frac{2}{\sqrt{\pi}}e^{-z}\sum_{n=0}\frac{2^n z^{2n+1}}{1.3..(2n+1)} \quad Then \quad erfc\left\{\frac{x}{\sqrt{2}}\right\}=1-O(xe^{-\frac{x}{\sqrt{2}}})$$

$$\left.\begin{array}{l}
DEFL^1\Big|i = DEFL^0\left\{\left[\dfrac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]\left[1-O(xe^{-\frac{x}{\sqrt{2}}})\right]+\dfrac{1}{4\gamma\rho_i\sqrt{r\pi}}\left[1-\left[1-O(\rho_i\sqrt{r}xe^{-\rho_i\sqrt{r}x})\right]\right]-\dfrac{\alpha_p^i\rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}}\left[1-e^{-\frac{x^2}{2}}\right]\right\} \\
= DEFL^0\left[\dfrac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]-O(xe^{-x})
\end{array}\right\}$$

$$DEFL^1\Big|i = DEFL^0\left\{\left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p}\right]erfc\left\{\frac{\tilde{\delta}_i^{\,p}}{\sqrt{2}\rho_i}\right\}+\frac{1}{2\gamma\rho_i\sqrt{r\pi}}\left[1-erfc\left\{\sqrt{r}\tilde{\delta}_i^{\,p}\right\}\right]-\frac{\alpha_p^i\rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}}\left[1-e^{-\frac{\left(\tilde{\delta}_i^{\,p}\right)^2}{2\rho_i^2}}\right]\right\}$$

$$\rho_i >> \tilde{\delta}_i^{\,p} \Rightarrow x = \frac{\tilde{\delta}_i^{\,p}}{\rho_i} << 1$$

$$DEFL^1\Big| i = DEFL^0 \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] erfc\left\{ \frac{x}{\sqrt{2}\rho_i} \right\} + \frac{1}{2\gamma\rho_i\sqrt{\pi r}} \left[1 - erfc\left\{ \rho_i \sqrt{r}\, x \right\} \right] - \frac{\alpha_p^i \rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}} \left[1 - e^{-\frac{x^2}{2}} \right] \right\}$$

$$erfc(x) = 1 - \frac{2}{\sqrt{\pi}} \sum_{n=0} \frac{(-1)^n z^{2n+1}}{n!(2n+1)} = 1 - \frac{2}{\sqrt{\pi}}(z - \frac{z^3}{3} ...)$$

$$DEFL^1\Big| i = DEFL^0 \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ 1 - \frac{2}{\sqrt{\pi}} \frac{x}{\sqrt{2}} + \frac{2}{3\sqrt{\pi}\sqrt{8}} x^3 - O(x^5) \right\} +$$

$$\frac{1}{2\gamma\rho_i\sqrt{\pi r}} \left[\frac{2}{\sqrt{\pi}} \rho_i \sqrt{r}\, x - \frac{2}{3\sqrt{\pi}} \left[\rho_i \sqrt{r} \right]^3 x^3 + O(x^5) \right] - \frac{\alpha_p^i \rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}} \left[1 - 1 + \frac{x^2}{2} - O(x^4) \right] =$$

$$DEFL^1\Big| i = DEFL^0 \left\{ \begin{array}{l} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \frac{2}{\sqrt{\pi}} \frac{x}{\sqrt{2}} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] + \frac{1}{2\gamma\rho_i\sqrt{\pi r}} \left[\frac{2}{\sqrt{\pi}} \rho_i \sqrt{r}\, x \right] - \\ - \frac{\alpha_p^i \rho_i}{2\gamma\hat{\sigma}_p\sqrt{2\pi}} \left[\frac{x^2}{2} \right] + \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ \frac{2}{3\sqrt{\pi}\sqrt{8}} x^3 \right\} - \frac{1}{2\gamma\rho_i\sqrt{\pi r}} \left[\frac{2}{3\sqrt{\pi}} \left[\rho_i \sqrt{r} \right]^3 x^3) \right] \end{array} \right\} - O(x^4)$$

$$DEFL^1\Big| i = DEFL^0 \left\{ \begin{array}{l} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \frac{2}{\sqrt{\pi}\sqrt{2}} \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \frac{1}{2\gamma\sqrt{\pi}} \right\} x + \\ - \frac{\alpha_p^i \rho_i}{4\gamma\hat{\sigma}_p\sqrt{2\pi}} x^2 + \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ \frac{2}{3\sqrt{\pi}\sqrt{8}} \right\} - \frac{\rho_i^2 r}{3\gamma\pi} \right\} x^3 \end{array} \right\} + O(x^4)$$

$$\gamma = \frac{1}{2\sqrt{2\pi}} \int_0^\infty \xi \, e^{-\xi^2} d\xi =$$

$$z = \xi^2 ; dz = 2\xi d\xi ; \xi d\xi = dz / 2$$

$$\gamma = \frac{1}{2\sqrt{2\pi}} \int_0^\infty \left[-\frac{e^{-z}}{2} \right] dz = \frac{1}{4\sqrt{2\pi}} = \frac{\sqrt{2}}{4\sqrt{\pi}} \, DEFL^0 \begin{Bmatrix} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \frac{2}{\sqrt{\pi}\sqrt{2}} \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \frac{1}{2\gamma\sqrt{\pi}} \right\} x + \\ - \frac{\alpha_p^i \rho_i}{4\gamma \hat{\sigma}_p \sqrt{2\pi}} x^2 + \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ \frac{2}{3\sqrt{\pi}\sqrt{8}} \right\} - \frac{\rho_i^2 r}{3\gamma\pi} \right\} x^3 \end{Bmatrix} + O$$

$$DEFL^1 \Big| i = DEFL^0 \begin{Bmatrix} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \sqrt{\frac{2}{\pi}} \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \frac{4\sqrt{\pi}}{2\sqrt{2}\sqrt{\pi}} \right\} x + \\ - \frac{\alpha_p^i \rho_i \, 4\sqrt{\pi}}{4\sqrt{2}\hat{\sigma}_p \sqrt{2\pi}} x^2 + \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ \frac{2}{3\sqrt{\pi}\sqrt{8}} \right\} - \frac{\rho_i^2 r 4\sqrt{\pi}}{3\sqrt{2}\pi} \right\} x^3 \end{Bmatrix} + O(x^4) =$$

$$= DEFL^0 \begin{Bmatrix} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] + \sqrt{\frac{2}{\pi}} \left\{ \sqrt{2} - \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \right\} x + \\ - \frac{\alpha_p^i \rho_i}{2\hat{\sigma}_p} x^2 + \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ \frac{2}{3\sqrt{\pi}\sqrt{8}} \sqrt{\frac{2}{2}} \right\} - \frac{2}{3} \sqrt{\frac{2}{\pi}} \rho_i^2 r \right\} x^3 \end{Bmatrix} + O(x^4) =$$

$$= DEFL^0 \begin{Bmatrix} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] + \sqrt{\frac{2}{\pi}} \left\{ \sqrt{2} - \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \right\} x + \\ - \frac{\alpha_p^i \rho_i}{2\hat{\sigma}_p} x^2 + \left\{ \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \left\{ \frac{1}{6} \sqrt{\frac{2}{\pi}} \right\} - \frac{2}{3} \sqrt{\frac{2}{\pi}} \rho_i^2 r \right\} x^3 \end{Bmatrix} + O(x^4) =$$

$$= DEFL^0 \begin{Bmatrix} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] + \sqrt{\frac{2}{\pi}} \left\{ \sqrt{2} - \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] \right\} x + \\ - \frac{\alpha_p^i \rho_i}{2\hat{\sigma}_p} x^2 + \frac{2}{3} \sqrt{\frac{2}{\pi}} \left\{ \frac{1}{4} \left[\frac{\hat{\sigma}_p^{R,i}}{\hat{\sigma}_p} \right] - \rho_i^2 r \right\} x^3 \end{Bmatrix} + O(x^4)$$